1 Introduction

Flavour physics provides an important opportunity for exploring the limits of the Standard Model of particle physics and for constraining possible extensions that go beyond it. As the LHC explores a new energy frontier and as experiments continue to extend the precision frontier, the importance of flavour physics will grow, both in terms of searches for signatures of new physics through precision measurements and in terms of attempts to construct the theoretical framework behind direct discoveries of new particles. A major theoretical limitation consists in the precision with which strong-interaction effects can be quantified. Large-scale numerical simulations of lattice QCD allow for the computation of these effects from first principles. The scope of the Flavour Lattice Averaging Group (FLAG) is to review the current status of lattice results for a variety of physical quantities in low-energy physics. Set up in November 2007 it comprises experts in Lattice Field Theory, Chiral Perturbation Theory and Standard Model phenomenology. Our aim is to provide an answer to the frequently posed question "What is currently the best lattice value for a particular quantity?" in a way that is readily accessible to nonlattice-experts. This is generally not an easy question to answer; different collaborations use different lattice actions (discretizations of QCD) with a variety of lattice spacings and volumes, and with a range of masses for the $u-$ and $d-$quarks. Not only are the systematic errors different, but also the methodology used to estimate these uncertainties varies between collaborations. In the present work we summarize the main features of each of the calculations and provide a framework for judging and combining the different results. Sometimes it is a single result that provides the "best" value; more often it is a combination of results from different collaborations. Indeed, the consistency of values obtained using different formulations adds significantly to our confidence in the results.

The first two editions of the FLAG review were published in 2011 [1] and 2014 [2]. The second edition reviewed results related to both light (u-, d- and s-), and heavy (c- and b-) flavours. The quantities related to pion and kaon physics were light-quark masses, the form factor $f_+(0)$ arising in semileptonic $K \to \pi$ transitions (evaluated at zero momentum transfer), the decay constants f_K and f_π, and the $B_{\rm K}$ parameter from neutral kaon mixing. Their implications for the CKM matrix elements V_{us} and V_{ud} were also discussed. Furthermore, results were reported for some of the low-energy constants of $SU(2)_L \times SU(2)_R$ and $SU(3)_L \times SU(3)_R$ Chiral Perturbation Theory. The quantities related to D- and B-meson physics that were reviewed were the B- and D-meson decay constants, form factors, and mixing parameters. These are the heavy-light quantities most relevant to the determination of CKM matrix elements and the global CKM unitarity-triangle fit. Last but not least, the current status of lattice results on the QCD coupling α_s was reviewed.

In the present paper we provide updated results for all the above-mentioned quantities, but also extend the scope of the review in two ways. First, we now present results for the charm and bottom quark masses, in addition to those of the three lightest quarks. Second, we review results obtained for the kaon mixing matrix elements of new operators that arise in theories of physics beyond the Standard Model. Our main results are collected in Tabs. 1 and 2.

Our plan is to continue providing FLAG updates, in the form of a peer reviewed paper, roughly on a biennial basis. This effort is supplemented by our more frequently updated website `http://itpwiki.unibe.ch/flag` [3], where figures as well as pdf-files for the individual sections can be downloaded. The papers reviewed in the present edition have appeared before the closing date **30 November 2015**.

Quantity	Sec.	$N_f = 2+1+1$	Refs.	$N_f = 2+1$	Refs.	$N_f = 2$	Refs.
m_s[MeV]	3.1.3	93.9(1.1)	[4, 5]	92.0(2.1)	[6–10]	101(3)	[11, 12]
m_{ud}[MeV]	3.1.3	3.70(17)	[4]	3.373(80)	[7–10, 13]	3.6(2)	[11]
m_s/m_{ud}	3.1.4	27.30(34)	[4, 14]	27.43(31)	[6–8, 10]	27.3(9)	[11]
m_u[MeV]	3.1.5	2.36(24)	[4]	2.16(9)(7)	‡	2.40(23)	[15]
m_d[MeV]	3.1.5	5.03(26)	[4]	4.68(14)(7)	‡	4.80(23)	[15]
m_u/m_d	3.1.5	0.470(56)	[4]	0.46(2)(2)	‡	0.50(4)	[15]
$\overline{m}_c$(3 GeV)[GeV]	3.2	0.996(25)	[4, 5]	0.987(6)	[9, 16]	1.03(4)	[11]
m_c/m_s	3.2.4	11.70(6)	[4, 5, 14]	11.82(16)	[16, 17]	11.74(35)	[11, 18]
$\overline{m}_b(\overline{m}_b)$[GeV]	3.3.4	4.190(21)	[5, 19]	4.164(23)	[9]	4.256(81)	[20, 21]
$f_+(0)$	4.3	0.9704(24)(22)	[22]	0.9677(27)	[23, 24]	0.9560(57)(62)	[25]
$f_{K^\pm}/f_{\pi^\pm}$	4.3	1.193(3)	[14, 26, 27]	1.192(5)	[28–31]	1.205(6)(17)	[32]
$f_{\pi^\pm}$[MeV]	4.6			130.2(1.4)	[28, 29, 31]		
$f_{K^\pm}$[MeV]	4.6	155.6(4)	[14, 26, 27]	155.9(9)	[28, 29, 31]	157.5(2.4)	[32]
$\Sigma^{1/3}$[MeV]	5.2.1	280(8)(15)	[33]	274(3)	[10, 13, 34, 35]	266(10)	[33, 36–38]
F_π/F	5.2.1	1.076(2)(2)	[39]	1.064(7)	[10, 29, 34, 35, 40]	1.073(15)	[36–38, 41]
$\bar{\ell}_3$	5.2.2	3.70(7)(26)	[39]	2.81(64)	[10, 29, 34, 35, 40]	3.41(82)	[36, 37, 41]
$\bar{\ell}_4$	5.2.2	4.67(3)(10)	[39]	4.10(45)	[10, 29, 34, 35, 40]	4.51(26)	[36, 37, 41]
$\bar{\ell}_6$	5.2.2					15.1(1.2)	[37, 41]
$\hat{B}_{\mathrm{K}}$	6.1	0.717(18)(16)	[42]	0.7625(97)	[10, 43–45]	0.727(22)(12)	[46]

‡ This is a FLAG estimate, based on χPT and the isospin averaged up- and down-quark mass m_{ud} [7–10, 13].

Table 1: Summary of the main results of this review, grouped in terms of N_f, the number of dynamical quark flavours in lattice simulations. Quark masses and the quark condensate are given in the $\overline{\mathrm{MS}}$ scheme at running scale $\mu = 2\,\mathrm{GeV}$ or as indicated; the other quantities listed are specified in the quoted sections. For each result we list the references that entered the FLAG average or estimate. From the entries in this column one can also read off the number of results that enter our averages for each quantity. We emphasize that these numbers only give a very rough indication of how thoroughly the quantity in question has been explored on the lattice and recommend to consult the detailed tables and figures in the relevant section for more significant information and for explanations on the source of the quoted errors.

Quantity	Sec.	$N_f = 2+1+1$	Refs.	$N_f = 2+1$	Refs.	$N_f = 2$	Refs.
f_D[MeV]	7.1	212.15(1.45)	[14, 27]	209.2(3.3)	[47, 48]	208(7)	[20]
f_{D_s}[MeV]	7.1	248.83(1.27)	[14, 27]	249.8(2.3)	[16, 48, 49]	250(7)	[20]
f_{D_s}/f_D	7.1	1.1716(32)	[14, 27]	1.187(12)	[47, 48]	1.20(2)	[20]
$f_+^{D\pi}(0)$	7.2			0.666(29)	[50]		
$f_+^{DK}(0)$	7.2			0.747(19)	[51]		
f_B[MeV]	8.1	186(4)	[52]	192.0(4.3)	[48, 53–56]	188(7)	[20, 57, 58]
f_{B_s}[MeV]	8.1	224(5)	[52]	228.4(3.7)	[48, 53–56]	227(7)	[20, 57, 58]
f_{B_s}/f_B	8.1	1.205(7)	[52]	1.201(16)	[48, 53–55]	1.206(23)	[20, 57, 58]
$f_{B_d}\sqrt{\hat{B}_{B_d}}$[MeV]	8.2			219(14)	[54, 59]	216(10)	[20]
$f_{B_s}\sqrt{\hat{B}_{B_s}}$[MeV]	8.2			270(16)	[54, 59]	262(10)	[20]
$\hat{B}_{B_d}$	8.2			1.26(9)	[54, 59]	1.30(6)	[20]
$\hat{B}_{B_s}$	8.2			1.32(6)	[54, 59]	1.32(5)	[20]
ξ	8.2			1.239(46)	[54, 60]	1.225(31)	[20]
B_{B_s}/B_{B_d}	8.2			1.039(63)	[54, 60]	1.007(21)	[20]
Quantity	Sec.	$N_f = 2+1$ and $N_f = 2+1+1$			Refs.		
$\alpha_{\overline{\rm MS}}^{(5)}(M_Z)$	9.9	0.1182(12)			[5, 9, 61–63]		
$\Lambda_{\overline{\rm MS}}^{(5)}$[MeV]	9.9	211(14)			[5, 9, 61–63]		

Table 2: Summary of the main results of this review, grouped in terms of N_f, the number of dynamical quark flavours in lattice simulations. The quantities listed are specified in the quoted sections. For each result we list the references that entered the FLAG average or estimate. From the entries in this column one can also read off the number of results that enter our averages for each quantity. We emphasize that these numbers only give a very rough indication of how thoroughly the quantity in question has been explored on the lattice and recommend to consult the detailed tables and figures in the relevant section for more significant information and for explanations on the source of the quoted errors.

This review is organized as follows. In the remainder of Sec. 1 we summarize the composition and rules of FLAG and discuss general issues that arise in modern lattice calculations. In Sec. 2 we explain our general methodology for evaluating the robustness of lattice results. We also describe the procedures followed for combining results from different collaborations in a single average or estimate (see Sec. 2.2 for our definition of these terms). The rest of the paper consists of sections, each dedicated to a single (or groups of closely connected) physical quantity(ies). Each of these sections is accompanied by an Appendix with explicatory notes.

1.1 FLAG composition, guidelines and rules

FLAG strives to be representative of the lattice community, both in terms of the geographical location of its members and the lattice collaborations to which they belong. We aspire to provide the particle-physics community with a single source of reliable information on lattice results.

In order to work reliably and efficiently, we have adopted a formal structure and a set of rules by which all FLAG members abide. The collaboration presently consists of an Advisory Board (AB), an Editorial Board (EB), and seven Working Groups (WG). The rôle of the Advisory Board is that of general supervision and consultation. Its members may interfere at any point in the process of drafting the paper, expressing their opinion and offering advice. They also give their approval of the final version of the preprint before it is rendered public. The Editorial Board coordinates the activities of FLAG, sets priorities and intermediate deadlines, and takes care of the editorial work needed to amalgamate the sections written by the individual working groups into a uniform and coherent review. The working groups concentrate on writing up the review of the physical quantities for which they are responsible, which is subsequently circulated to the whole collaboration for critical evaluation.

The current list of FLAG members and their Working Group assignments is:

- Advisory Board (AB): S. Aoki, C. Bernard, M. Golterman, H. Leutwyler, and C. Sachrajda
- Editorial Board (EB): G. Colangelo, A. Jüttner, S. Hashimoto, S. Sharpe, A. Vladikas, and U. Wenger
- Working Groups (coordinator listed first):
  - Quark masses — L. Lellouch, T. Blum, and V. Lubicz
  - V_{us}, V_{ud} — S. Simula, P. Boyle,[1] and T. Kaneko
  - LEC — S. Dürr, H. Fukaya, and U.M. Heller
  - B_K — H. Wittig, P. Dimopoulos, and R. Mawhinney
  - $f_{B_{(s)}}$, $f_{D_{(s)}}$, B_B — M. Della Morte, Y. Aoki, and D. Lin
  - $B_{(s)}$, D semileptonic and radiative decays — E. Lunghi, D. Becirevic, S. Gottlieb, and C. Pena
  - α_s — R. Sommer, R. Horsley, and T. Onogi

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[1] Peter Boyle had participated actively in the early stages of the current FLAG effort. Unfortunately, due to other commitments, it was impossible for him to contribute until the end, and he decided to withdraw from the collaboration.

As some members of the WG on quark masses were faced with unexpected hindrances, S. Simula has kindly assisted in the completion of the relevant section during the final phases of its composition.

The most important FLAG guidelines and rules are the following:

- the composition of the AB reflects the main geographical areas in which lattice collaborations are active, with members from America, Asia/Oceania and Europe;
- the mandate of regular members is not limited in time, but we expect that a certain turnover will occur naturally;
- whenever a replacement becomes necessary this has to keep, and possibly improve, the balance in FLAG, so that different collaborations, from different geographical areas are represented;
- in all working groups the three members must belong to three different lattice collaborations;[2]
- a paper is in general not reviewed (nor colour-coded, as described in the next section) by any of its authors;
- lattice collaborations not represented in FLAG will be consulted on the colour coding of their calculation;
- there are also internal rules regulating our work, such as voting procedures.

1.2 Citation policy

We draw attention to this particularly important point. As stated above, our aim is to make lattice QCD results easily accessible to nonlattice-experts and we are well aware that it is likely that some readers will only consult the present paper and not the original lattice literature. It is very important that this paper be not the only one cited when our results are quoted. We strongly suggest that readers also cite the original sources. In order to facilitate this, in Tabs. 1 and 2, besides summarizing the main results of the present review, we also cite the original references from which they have been obtained. In addition, for each figure we make a bibtex-file available on our webpage [3] which contains the bibtex-entries of all the calculations contributing to the FLAG average or estimate. The bibliography at the end of this paper should also make it easy to cite additional papers. Indeed we hope that the bibliography will be one of the most widely used elements of the whole paper.

1.3 General issues

Several general issues concerning the present review are thoroughly discussed in Sec. 1.1 of our initial 2010 paper [1] and we encourage the reader to consult the relevant pages. In the remainder of the present subsection, we focus on a few important points. Though the discussion has been duly updated, it is essentially that of Sec. 1.2 of the 2013 review [2].

The present review aims to achieve two distinct goals: first, to provide a **description** of the work done on the lattice concerning low-energy particle physics; and, second, to draw

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[2] The WG on semileptonic D and B decays has currently four members, but only three of them belong to lattice collaborations.

conclusions on the basis of that work, summarizing the results obtained for the various quantities of physical interest.

The core of the information about the work done on the lattice is presented in the form of tables, which not only list the various results, but also describe the quality of the data that underlie them. We consider it important that this part of the review represents a generally accepted description of the work done. For this reason, we explicitly specify the quality requirements[3] used and provide sufficient details in appendices so that the reader can verify the information given in the tables.

On the other hand, the conclusions drawn on the basis of the available lattice results are the responsibility of FLAG alone. Preferring to err on the side of caution, in several cases we draw conclusions that are more conservative than those resulting from a plain weighted average of the available lattice results. This cautious approach is usually adopted when the average is dominated by a single lattice result, or when only one lattice result is available for a given quantity. In such cases one does not have the same degree of confidence in results and errors as when there is agreement among several different calculations using different approaches. The reader should keep in mind that the degree of confidence cannot be quantified, and it is not reflected in the quoted errors.

Each discretization has its merits, but also its shortcomings. For most topics covered in this review we have an increasingly broad database, and for most quantities lattice calculations based on totally different discretizations are now available. This is illustrated by the dense population of the tables and figures in most parts of this review. Those calculations that do satisfy our quality criteria indeed lead to consistent results, confirming universality within the accuracy reached. In our opinion, the consistency between independent lattice results, obtained with different discretizations, methods, and simulation parameters, is an important test of lattice QCD, and observing such consistency also provides further evidence that systematic errors are fully under control.

In the sections dealing with heavy quarks and with α_s, the situation is not the same. Since the b-quark mass cannot be resolved with current lattice spacings, all lattice methods for treating b quarks use effective field theory at some level. This introduces additional complications not present in the light-quark sector. An overview of the issues specific to heavy-quark quantities is given in the introduction of Sec. 8. For B and D meson leptonic decay constants, there already exists a good number of different independent calculations that use different heavy-quark methods, but there are only one or two independent calculations of semileptonic B and D meson form factors and B meson mixing parameters. For α_s, most lattice methods involve a range of scales that need to be resolved and controlling the systematic error over a large range of scales is more demanding. The issues specific to determinations of the strong coupling are summarized in Sec. 9.

Number of sea quarks in lattice simulations:
Lattice QCD simulations currently involve two, three or four flavours of dynamical quarks. Most simulations set the masses of the two lightest quarks to be equal, while the strange and charm quarks, if present, are heavier (and tuned to lie close to their respective physical values). Our notation for these simulations indicates which quarks are nondegenerate, e.g. $N_f = 2+1$ if $m_u = m_d < m_s$ and $N_f = 2+1+1$ if $m_u = m_d < m_s < m_c$. Calculations with $N_f = 2$, i.e. two degenerate dynamical flavours, often include strange valence quarks

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[3] We also use terms like "quality criteria", "rating", "colour coding" etc. when referring to the classification of results, as described in Sec. 2.

interacting with gluons, so that bound states with the quantum numbers of the kaons can be studied, albeit neglecting strange sea-quark fluctuations. The quenched approximation ($N_f = 0$), in which sea quark contributions are omitted, has uncontrolled systematic errors and is no longer used in modern lattice simulations with relevance to phenomenology. Accordingly, we will review results obtained with $N_f = 2$, $N_f = 2+1$, and $N_f = 2+1+1$, but omit earlier results with $N_f = 0$. The only exception concerns the QCD coupling constant α_s. Since this observable does not require valence light quarks, it is theoretically well defined also in the $N_f = 0$ theory, which is simply pure gluon-dynamics. The N_f-dependence of α_s, or more precisely of the related quantity $r_0\Lambda_{\overline{\rm MS}}$, is a theoretical issue of considerable interest; here r_0 is a quantity with the dimension of length, which sets the physical scale, as discussed in Appendix A.2. We stress, however, that only results with $N_f \geq 3$ are used to determine the physical value of α_s at a high scale.

Lattice actions, simulation parameters and scale setting:
The remarkable progress in the precision of lattice calculations is due to improved algorithms, better computing resources and, last but not least, conceptual developments. Examples of the latter are improved actions that reduce lattice artifacts and actions that preserve chiral symmetry to very good approximation. A concise characterization of the various discretizations that underlie the results reported in the present review is given in Appendix A.1.

Physical quantities are computed in lattice simulations in units of the lattice spacing so that they are dimensionless. For example, the pion decay constant that is obtained from a simulation is $f_\pi a$, where a is the spacing between two neighboring lattice sites. To convert these results to physical units requires knowledge of the lattice spacing a at the fixed values of the bare QCD parameters (quark masses and gauge coupling) used in the simulation. This is achieved by requiring agreement between the lattice calculation and experimental measurement of a known quantity, which thus “sets the scale” of a given simulation. A few details on this procedure are provided in Appendix A.2.

Renormalization and scheme dependence:
Several of the results covered by this review, such as quark masses, the gauge coupling, and B-parameters, are for quantities defined in a given renormalization scheme and at a specific renormalization scale. The schemes employed (e.g. regularization-independent MOM schemes) are often chosen because of their specific merits when combined with the lattice regularization. For a brief discussion of their properties, see Appendix A.3. The conversion of the results, obtained in these so-called intermediate schemes, to more familiar regularization schemes, such as the $\overline{\rm MS}$-scheme, is done with the aid of perturbation theory. It must be stressed that the renormalization scales accessible in simulations are limited, because of the presence of an ultraviolet (UV) cutoff of $\sim \pi/a$. To safely match to $\overline{\rm MS}$, a scheme defined in perturbation theory, Renormalization Group (RG) running to higher scales is performed, either perturbatively or nonperturbatively (the latter using finite-size scaling techniques).

Extrapolations:
Because of limited computing resources, lattice simulations are often performed at unphysically heavy pion masses, although results at the physical point have become increasingly common. Further, numerical simulations must be done at nonzero lattice spacing, and in a finite (four- dimensional) volume. In order to obtain physical results, lattice data are obtained at a sequence of pion masses and a sequence of lattice spacings, and then extrapolated to the physical pion mass and to the continuum limit. In principle, an extrapolation to infinite volume is also required. However, for most quantities discussed in this review, finite-volume

effects are exponentially small in the linear extent of the lattice in units of the pion mass and, in practice, one often verifies volume independence by comparing results obtained on a few different physical volumes, holding other parameters equal. To control the associated systematic uncertainties, these extrapolations are guided by effective theories. For light-quark actions, the lattice-spacing dependence is described by Symanzik's effective theory [64, 65]; for heavy quarks, this can be extended and/or supplemented by other effective theories such as Heavy-Quark Effective Theory (HQET). The pion-mass dependence can be parameterized with Chiral Perturbation Theory (χPT), which takes into account the Nambu-Goldstone nature of the lowest excitations that occur in the presence of light quarks. Similarly, one can use Heavy-Light Meson Chiral Perturbation Theory (HMχPT) to extrapolate quantities involving mesons composed of one heavy (b or c) and one light quark. One can combine Symanzik's effective theory with χPT to simultaneously extrapolate to the physical pion mass and the continuum; in this case, the form of the effective theory depends on the discretization. See Appendix A.4 for a brief description of the different variants in use and some useful references. Finally, χPT can also be used to estimate the size of finite-volume effects measured in units of the inverse pion mass, thus providing information on the systematic error due to finite-volume effects in addition to that obtained by comparing simulations at different volumes.

Critical slowing down:
The lattice spacings reached in recent simulations go down to 0.05 fm or even smaller. In this regime, long autocorrelation times slow down the sampling of the configurations [66–75]. Many groups check for autocorrelations in a number of observables, including the topological charge, for which a rapid growth of the autocorrelation time is observed with decreasing lattice spacing. This is often referred to as topological freezing. A solution to the problem consists in using open boundary conditions in time, instead of the more common antiperiodic ones [76]. More recently two other approaches have been proposed, one based on a multiscale thermalization algorithm [77] and another based on defining QCD on a nonorientable manifold [78]. The problem is also touched upon in Sec. 9.2, where it is stressed that attention must be paid to this issue. While large scale simulations with open boundary conditions are already far advanced [79], unfortunately so far no results reviewed here have been obtained with any of the above methods. It is usually *assumed* that the continuum limit can be reached by extrapolation from the existing simulations and that potential systematic errors due to the long autocorrelation times have been adequately controlled.

Simulation algorithms and numerical errors:
Most of the modern lattice-QCD simulations use exact algorithms such as those of Refs. [80, 81], which do not produce any systematic errors when exact arithmetic is available. In reality, one uses numerical calculations at double (or in some cases even single) precision, and some errors are unavoidable. More importantly, the inversion of the Dirac operator is carried out iteratively and it is truncated once some accuracy is reached, which is another source of potential systematic error. In most cases, these errors have been confirmed to be much less than the statistical errors. In the following we assume that this source of error is negligible. Some of the most recent simulations use an inexact algorithm in order to speed-up the computation, though it may produce systematic effects. Currently available tests indicate that errors from the use of inexact algorithms are under control.

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